JSPM's

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MHT- CET 2016 Solution

Subject :- Mathematics

1 : (B)

Here,

E(x) = nP = 5 and Var (x) = npq = 2.5

$$\therefore \quad q = \frac{2.5}{5} = \frac{1}{2}$$

$$\therefore \quad p = \frac{1}{2} \text{ and } n = 10$$

Now, $P(x<1) = P(x=0)$

$$\Rightarrow P(x<1) = {}^{10}C_0 \left(\frac{1}{2}\right)^{10}$$

$$\Rightarrow P(x<1) = \left(\frac{1}{2}\right)^{10}$$

2 : (D)

Correct option is (D)

3: (B)

As required circle touches y-axis at the origin.

$\therefore$ Let Centre of the circle is d (a, 0) and radius is a

$\therefore$ Equation of circle will be,

$$(x-a)^2 + (y-0)^2 = a^2$$

$$\Rightarrow \quad x^2 - 2ax + a^2 + y^2 = a^2$$

$$\Rightarrow \quad x^2 + y^2 - 2ax = 0 \qquad \text{...(i)}$$

By differentiating above equation w.r.t. x, we get

$$2x+2y\frac{dy}{dx}-2a=0$$

$$\therefore \quad 2a=2x+2y\frac{dy}{dx} \qquad \text{...(ii)}$$

From (i) and (ii),

$$x^2+y^2-\left(2x+2y\frac{dy}{dx}\right)x=0$$

$$\Rightarrow \quad x^2+y^2-2x^2-2xy\frac{dy}{dx}=0$$

$$\Rightarrow \quad x^2-y^2+2xy\frac{dy}{dx}=0$$

4 : (B)

Here,

a_{ij} is stands for element of matrix A as i^{th} row and j^{th} column, and A_{ij} stands for co-factor of element a_{ij} of matrix A.

$\therefore \quad a_{11}=1, \quad a_{12}=1$ and $a_{13}=0$

And

$$A_{21}=(-1)^{2+1}\begin{vmatrix}1 & 0\\ 2 & 1\end{vmatrix}=-1$$

$$A_{22}=(-1)^{2+2}\begin{vmatrix}1 & 0\\ 1 & 1\end{vmatrix}=1$$

$$A_{23}=(-1)^{2+3}\begin{vmatrix}1 & 1\\ 1 & 2\end{vmatrix}=-1$$

Therefore

$$a_{11}A_{21}+a_{12}A_{22}+a_{13}A_{23}=1\times(-1)+1\times(1)+0\times(-1)=0$$

5 : (D)

Given, $f(x) = e^x(\sin x - \cos x)$

$\therefore \quad f'(x) = e^x[\cos x + \sin x] + [\sin x - \cos x]e^x$

$\therefore \quad f'(x) = 2e^x \sin x$

To verify Rolle's Theorem.

$f'(c) = 0$

$2e^c \sin c = 0$

$\Rightarrow \quad \sin c = 0$

$\therefore \quad c = \pi$

6 : (C)

As given, both line passes through (0, 0), and $\theta_1 = \frac{\pi}{6}, \quad \theta_2 = \frac{\pi}{3}$

$\therefore$ Equation of first line is.

$$y - 0 = \tan\left(\frac{\pi}{6}\right)(x - 0)$$

$$\Rightarrow \quad y = \frac{1}{\sqrt{3}}x$$

$$\Rightarrow \quad x - \sqrt{3}y = 0 \qquad \text{...(i)}$$

Equation of second line is

$$y - 0 = \tan\left(\frac{\pi}{3}\right)(x - 0)$$

$$\Rightarrow \quad y = \sqrt{3}\,x$$

$$\Rightarrow \quad \sqrt{3}\,x - y = 0 \qquad \text{...(ii)}$$

Hence, joint equation of these line is

$$(x-\sqrt{3}y)(\sqrt{3}x-y)=0$$

$$\Rightarrow \sqrt{3}x^2-xy-3xy+\sqrt{3}y^2=0$$

$$\Rightarrow \sqrt{3}x^2+\sqrt{3}y^2-4xy=0$$

7 : (B)

As given, $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

$$\tan^{-1}(\cos x)+\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$$

$$=\tan^{-1}\left[\frac{2\cos x}{1-\cos^2 x}\right]=\tan^{-1}(2\operatorname{cosec} x)$$

$$\therefore \quad \frac{2\cos x}{\sin^2 x}=2\operatorname{cosec} x$$

$$\therefore \quad 2\cot x=2$$

$$\therefore \quad \cot x=1$$

$$\therefore \quad x=\frac{\pi}{4}$$

Hence,

$$\sin x+\cos x=\sin\frac{\pi}{4}+\cos\frac{\pi}{4}$$

$$=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}$$

$$=\sqrt{2}$$

8 : (A)

Option (A) is correct.

9 : (D)

Let $I = \int \frac{dx}{\sqrt{8+2x-x^2}}$

$\Rightarrow I = \int \frac{dx}{9+2x-x^2-1}$

$\Rightarrow I = \int \frac{dx}{3^2-(x-1)^2}$

$\therefore I = \sin^{-1}\left(\frac{x-1}{3}\right)+c$

10 : (A)

As given,

$f(x) = x^3+5x^2-7x+9$

$\therefore \quad f(1.1) = 8.6$

11 : (B)

As given,

$$f(x) = \begin{cases} \frac{1}{5}, & 0 \le x \le 5 \\ 0, & otherwise \end{cases}$$

Now, probability of waiting time not more than 4 is $= 4 \times \frac{1}{5} = 0.8$

12 : (B)

Let $X=(a-b)^2\cos^2\frac{c}{2}+(a+b)^2\sin^2\frac{c}{2}$

As we know,

$\sin\frac{c}{2}=\sqrt{\frac{(s-a)(s-b)}{ab}}$ and $\cos\frac{c}{2}=\sqrt{\frac{s(s-c)}{ab}}$

Where $s=\frac{a+b+c}{2}$

By substituting these value on above equation we will get

$X=c^2$

13 : (B)

Let $y_1=\log(\sec\theta+\tan\theta)$

$\therefore\ \frac{dy_1}{d\theta}=\frac{1}{\sec\theta+\tan\theta}\cdot(\sec\theta\tan\theta+\sec^2 a)$

$\Rightarrow\ \frac{dy_1}{d\theta}=\frac{\sec\theta[\sec\theta+\tan\theta]}{[\sec\theta+\tan\theta]}$

$\Rightarrow\ \frac{dy_1}{d\theta}=\sec\theta$ ....(i)

Now,

Let $y_2=\sec\theta$

$\therefore\ \frac{dy_2}{d\theta}=\sec\theta\cdot\tan\theta$ ...(ii)

$\therefore\ \frac{dy_1}{dy_2}=\frac{\sec\theta}{\sec\theta\cdot\tan\theta}=\cot\theta$

$$\therefore \quad \left.\frac{dy_1}{dy_2}\right|_{\theta=\frac{\pi}{4}} = \cot\frac{\pi}{4} = 1$$

14 : (B)

We can say that both line passes through point (5, 3) and makes angle 45° and 135° with x axis

$\therefore$ Equation of first line is ,

$$y-3=\tan 45^{o}\ (x-5)$$

$$y-3=x-5$$

$$y-x+2=0 \qquad \text{...(i)}$$

Similarly,

$$y-3=\tan 135^{o}\,(x-5)$$

$$y-3=-1(x-5)$$

$$y+x-8=0 \qquad \text{....(ii)}$$

$\therefore$ Joint equation of line is

$$(y-x+2)(y+x-8)=0$$

$$\Rightarrow \quad x^2-y^2-10x+6y+16=0$$

15 : (A)

As given, required point is on the

Curve $6y=x^3+2$

Therefore, only point (4, 11)

Satisfy the given equation,

Hence, option (A) is Correct.

16 : (A)

$$f(x)=\begin{cases} x\sin\frac{1}{x} & for \quad x\neq 0 \\ k & for \quad x=0 \end{cases}$$

$$\therefore \lim_{x\to 0} x \ \sin\frac{1}{x}=k$$

$$\Rightarrow 0=k$$

17 : (C)

Given, $y=e^{m\sin^{-1}x}$

$$\Rightarrow \quad \frac{dy}{dx}=e^{m\sin^{-1}x}\cdot m\cdot\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)^2=\frac{m^2y^2}{1-x^2}$$

$$\therefore \quad (1-x^2)\left(\frac{dy}{dx}\right)^2=m^2y^2$$

$$\therefore \quad A=m^2$$

18 : (B)

Let $I=\int\left(\frac{4e^x-25}{2e^x-5}\right)dx$

$$\Rightarrow I=\int\left(\frac{10e^x-25-6e^x}{2e^x-5}\right)dx$$

$$\Rightarrow I=\int\frac{5(2e^x-5)-6e^x}{2e^x-5}dx$$

$$\Rightarrow I=\int\left(5-\frac{6e^x}{2e^x-5}\right)dx$$

$$\Rightarrow I = 5x - 3\log(2e^x - 5) + C$$

$$\therefore \ A = 5 \text{ and } B = -3$$

19 : (B)

By solving we will get,

$$\frac{\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)}{\operatorname{cosec}^{-1}(-\sqrt{2}) + \cos^{-1}\left(-\frac{1}{2}\right)} = -\frac{4}{5}$$

20 : (C)

As given, $f(x) = \begin{cases} \dfrac{\log(1+2x)\sin x^\circ}{x^2} & \text{for } x \neq 0 \\ k & \text{for } x = 0 \end{cases}$

Is continuous at x = 0

$$\therefore \ \lim_{x\to 0}\frac{\log(1+2x)\cdot\sin x^\circ}{x^2} = k$$

$$\therefore \ \lim_{x\to 0}\frac{2\cdot\log(1+2x)}{2x}\cdot\lim_{x\to 0}\frac{\sin\left(x\times\frac{\pi}{180}\right)}{\left(x\times\frac{\pi}{180}\right)}\times\frac{\pi}{180} = k$$

$$\Rightarrow \quad 2\times\frac{\pi}{180} = k$$

$$\Rightarrow \quad \frac{\pi}{90} = k$$

21 : (A)

Given, $\log_{10}\left(\dfrac{x^2 - y^2}{x^2 + y^2}\right) = 2$

$$\Rightarrow \quad \frac{x^2 - y^2}{x^2 + y^2} = 100$$

$$\Rightarrow \quad \frac{x^2 - y^2 - 100x^2 - 100y^2}{x^2 + y^2} = 0$$

$$\Rightarrow \quad \frac{-99x^2 - 101y^2}{x^2 + y^2} = 0$$

$$\Rightarrow \quad 99x^2 + 101y^2 = 0$$

$$\Rightarrow \quad (2 \times 99)x + (2 \times 101)y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-99x}{101y}$$

22 : (D)

Let $I = \int_{-\pi/2}^{\pi/2} \log\left(\frac{2 - \sin x}{2 + \sin x}\right) dx$

As given function is odd.

$\therefore \quad \int I = 0$

23 : (C)

By using anti differentiation method,

We will get to know that, Option (C) is correct.

24 : (B)

Degree $\Rightarrow \; 3$

Order $\Rightarrow \; 2$

25 : (B)

Acute angle is $\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)$

26 : (B)

$$A=\int_0^2 (2x-x^2)dx$$

$$\Rightarrow A=\left[x^2-\frac{x^3}{3}\right]_0^2$$

$$\Rightarrow A=\left[4-\frac{8}{3}\right]$$

$\Rightarrow A=\frac{4}{3}$ sq unit.

27 : (A)

Given $\int \frac{f(x)}{\log(\sin x)}dx=\log[\log\sin x]+C$

By using anti differentiation method,we will get

$$\frac{d}{dx}(\log[\log\sin x]+c)$$

$$=\frac{1}{\log(\sin x)}\cdot\frac{1}{\sin x}\cdot\cos x$$

$$=\frac{\cot x}{\log(\sin x)}$$

$$\therefore \quad f(x)=\cot x$$

28 : (A)

Correct option is (A)

29 : (C)

$$3\overline{i}-\overline{k}=m\overline{i}+m\overline{j}-2m\overline{k}+2n\overline{i}-n\overline{j}+n\overline{k}$$

$$\Rightarrow\ 3\overline{i}-\overline{k}=(m+2n)\overline{i}+(m-n)\overline{i}+(n-2n)\overline{k}$$

$\therefore\quad m+2n=3$ ...(i)

$m-n=0$ ....(ii)

$n-2n=-1$ ...(iii)

As $m-n=0$ from (ii)

$\therefore\quad m=n$

$\therefore\quad 3m=3$

$\therefore\quad m=1$ and n = 1

$\therefore\quad m+n=2$

30 : (C)

Let

$$I=\int_0^{\pi/2}\frac{\sqrt[n]{\sec x}}{\sqrt[n]{\sec x}+\sqrt[n]{\operatorname{cosec} x}}dx \qquad \text{...(i)}$$

$$\Rightarrow\ I=\int_0^{\pi/2}\frac{\sqrt[n]{\sec\left(\frac{\pi}{2}-x\right)}}{\sqrt[n]{\sec\left(\frac{\pi}{2}-x\right)}+\sqrt[n]{\operatorname{cosec}\left(\frac{\pi}{2}-x\right)}}dx$$

$$\Rightarrow\ I=\int_0^{\pi/2}\frac{\sqrt[n]{\operatorname{cosec} x}}{\sqrt[n]{\operatorname{cosec} x}+\sqrt[n]{\sec x}}dx \qquad \text{...(ii)}$$

Adding equation (i) and (ii)

$$2I = \int_0^{\pi/2} dx$$

$$\Rightarrow 2I = [x]_0^{\pi/2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

31 : (C)

Correct option is (C)

32 : (A)

$$y(1+\log x)\frac{dx}{dy} - x \log x = 0$$

$$\Rightarrow \left(\frac{1+\log x}{x \log x}\right)dx = \frac{dy}{y}$$

Integrating on both side

$$\int\left(\frac{1+\log x}{x \log x}\right)dx = \int \frac{dy}{y}$$

$$\log(x \log x) = \log y + \log C$$

$$\Rightarrow \log(x \log x) = \log(y \cdot c)$$

$$\therefore \quad x \log x = y \cdot c \qquad \text{...(i)}$$

As $x = e, \quad y = e^2$

$$\therefore \quad e = e^2 \cdot c$$

$$\therefore \quad c = \frac{1}{e}$$

Putting $c = \frac{1}{e}$ in eq (i) we get

$x \log x = \frac{y}{e}$

$\Rightarrow \quad y = ex \log x$

33 : (C)

Correct option is (C)

34 : (D)

Given.

I.F of $\frac{dy}{dx} + py = Q$ is $\sin x$

$\therefore \quad e^{\int P\,dx} = \sin x$

$\Rightarrow \quad \int P\,dx = \ln(\sin x)$

By anti-differentiation method, we will get

$P = \frac{d}{dx}[\ln(\sin x)]$

$= \frac{1}{\sin x} \cdot \cos x$

$P = \cot x$

35 : (C)

Option (C) is the correct answer

36 : (A)

$P(x \geq 7) = P(x=7) + P(x=8) + P(x=9) + P(x=10)$

$$= {}^{10}C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 + {}^{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + {}^{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^1 + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$$

$$= \frac{11}{64}$$

37 : (C)

Given,

$$\sin 2x + \cos 2x = 0$$

Multiplying by $\frac{1}{\sqrt{2}}$ on both side,

$$\therefore \quad \frac{1}{\sqrt{2}}\sin 2x + \frac{1}{\sqrt{2}}\cos 2x = 0$$

$$\Rightarrow \quad \sin\left(\frac{\pi}{4} + 2x\right) = 0$$

$$\Rightarrow \quad 2x + \frac{\pi}{4} = n\pi$$

$$\Rightarrow \quad x = \frac{(4n-1)\pi}{8}$$

$$\therefore \quad x = \frac{11\pi}{8} \quad and \quad \frac{15\pi}{8}$$

38 : (A)

Correct option is (A)

39 : (A)

Given $A = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$ *and* $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$$\therefore \quad A^{-1} = \frac{1}{10}\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} \quad and \quad B^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Now,

$$B^{-1} \cdot A^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \frac{1}{10}\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 3 & 2 \\ -2 & 2 \end{bmatrix}$$

$$(B^{-1} \cdot A^{-1})^{-1} = \frac{10}{10}\begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$$

40 : (D)

p : Every square is a rectangle $\rightarrow$ T

q : Every rhombus is a kite $\rightarrow$ T

$$\therefore \quad p \rightarrow q \equiv T$$

$$p \leftrightarrow q \equiv T$$

41 : (B)

Correct option is (B)

42 : (A)

Correct option is (A)

43 : (C)

$$\tan^2 x = 1$$

$$\therefore \quad \tan x = \pm 1$$

$= x = n\pi \pm \frac{\pi}{4}$

44 : (B)

Correct option is (B)

45 : (B)

Correct option is (B)

46 : (C)

Given $A\ x = I$

$\therefore \quad x = A^{-1}$

$\therefore \quad x = -\frac{1}{5}\begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix}$

$\therefore \quad x = \frac{1}{5}\begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix}$

47 : (A)

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & \lambda & 1 \\ 1 & -1 & 4 \end{vmatrix} = 10$$

$1(4\lambda + 1) - 1(8 - 1) + 1(-2 - \lambda) = 10$

$\Rightarrow \quad 4\lambda + 1 - 7 - 2 - \lambda = 10$

$\Rightarrow \quad 3\lambda = 18$

$\Rightarrow \quad \lambda = 6$

48 : (B)

Given

n = 5

$p = \frac{1}{3}$

$\therefore \quad q = \frac{2}{3}$

$p(2 < x < 4) = p(x = 3)$

$= {}^5C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2$

$= \frac{5 \times 4}{2} \times \frac{1}{27} \times \frac{4}{9}$

$= \frac{40}{243}$

49 : (C)

Correct option is (C)

50 : (D)

$s_1 \equiv p$

$s_2 \equiv q$