JSPM's

Imperial College of Engineering and Research, Wagholi, Pune.

(Approved by AICTE, Delhi & Govt. of Maharashtra, affiliated to SPPU)

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Accredited with 'A' Grade by NAAC

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MHT- CET 2017 Solution

Subject :- Mathematics

1. The number of principal solutions of $\tan 2\theta = 1$ is

(A) one (B) Two (C) Three (D) Four

1. **(B)**

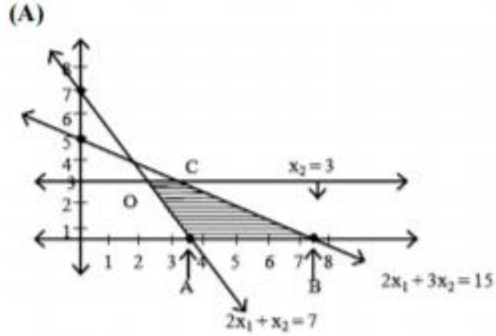
$\tan 2\theta = 1(+\text{ive})$

1^{st} and 3^{rd} quadrant

2. The objective function $x = 4x_1 + 5x_2$, subject to $2x_1 + x_2 \geq 7, 2x_1 + 3x_2 \leq 15, x_2 \leq 3, x_1, x_2 \geq 0$ has minimum value at the point.

(A) On x-axis (B) On y-axis (C) At the origin (D) On the line parallel to x-axis

2. **(A)**



Corner point

Value of $z = 4x_1 + 5x_2$

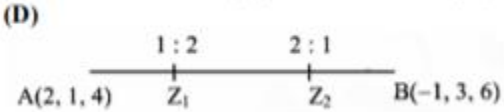
Since two points are on x-axis minimum value occurs on x-axis.

Minimum value = 14.

3. If z_1 and z_2 are z co-ordinates of the point of trisection of the segment joining the points $A(2,1,4), B(-1,3,6)$ then $z_1 + z_2 =$

(A) 1 (B) 4 (C) 5 (D) 10

3. **(D)**



For $Z_1 \to 1:2$

For $Z_2 \to 2:1$

(Internal division formula)

$Z = Z_1 + Z_2$

$= \frac{(1)(6)+2(4)}{1+2} + \frac{2(6)+(1)(4)}{2+1}$

$= \frac{6+8}{3} + \frac{12+4}{3}$

$= \frac{14+16}{3}$

$= \frac{30}{3} \qquad = 10$

4. The maximum value of $f(x)=\frac{\log x}{x}(x\neq 0, x\neq 1)$ is

(A) e (B) $\frac{1}{e}$ (C) e^2 (D) $\frac{1}{e^2}$

4. **(B)**

$f(x)=\frac{\log x}{x}$

$f'(x)=\frac{x.\frac{1}{x}-\log x}{x^2}=\frac{1-\log x}{x^2}$

$f'(x)=0\Rightarrow \frac{1-\log x}{x^2}=0$

$\text{Log}\, x=1$

$x=e$

max. value $f(e)=\frac{\log e}{e}=\frac{1}{e}$

5. $\int_0^1 x\tan^{-1}x\,dx=$

(A) $\frac{\pi}{4}+\frac{1}{2}$ (B) $\frac{\pi}{4}-\frac{1}{2}$ (C) $\frac{1}{2}-\frac{\pi}{4}$ (D) $-\frac{\pi}{4}-\frac{1}{2}$

5. **(B)**

$$\int_0^1 x\tan^{-1}x\,dx=\left[\tan^{-1}x\int x\,dx\right]_0^1-\int_0^1\left(\frac{d}{dx}\tan^{-1}x\int x\,dx\right)dx$$

$$=\left(\tan^{-1}x\cdot\frac{x^2}{2}\right)_0^1-\int_0^1\frac{1}{1+x^2}\cdot\frac{x^2}{2}dx$$

$$=\left(\frac{\pi}{4}\cdot\frac{1}{2}-0\right)-\frac{1}{2}\int_0^1\frac{1+x^2-1}{1+x^2}dx$$

$$=\frac{\pi}{8}-\frac{1}{2}\int_0^1\left(1-\frac{1}{1+x^2}\right)dx$$

$$=\frac{\pi}{8}-\frac{1}{2}\left[x-\tan^{-1}x\right]_0^1$$

$$=\frac{\pi}{8}-\frac{1}{2}\left[(1-0)-\left(\frac{\pi}{4}-0\right)\right]=\frac{\pi}{8}-\frac{1}{2}+\frac{\pi}{8}=\frac{\pi}{4}-\frac{1}{2}$$

6. The statement pattern $(\sim p\wedge q)$ is logically equivalent to

(A) $(p\vee q)\vee\sim p$ (B) $(p\vee q)\wedge\sim p$ (C) $(p\wedge q)\rightarrow p$ (D) $(p\vee q)\rightarrow p$

6. **(B)**

$(\sim p\wedge q)$ is logically equivalent to

$A\rightarrow(p\vee q)\vee\sim p \quad = \quad T\vee q=T$

$B \rightarrow (p \vee q) \wedge \sim p \quad = \quad (p \wedge \sim p) \vee (q \wedge \sim p)$ ... Distributive law

$= \quad F \vee (q \wedge \sim p)$ ... Complementary law

$= \quad q \wedge \sim p$ ... Identify law

$= \quad \sim p \wedge q$ ...Commutative law

7. If g(x) is the inverse function of f(x) and $f'(x) = \frac{1}{1+x^4}$ then $g'(x)$ is

(A) $1+[g(x)]^4$ (B) $1-[g(x)]^4$ (C) $1+[f(x)]^4$ (D) $\frac{1}{1+[g(x)]^4}$

7. **(A)**

$g = f^{-1}$

$f(g(x)) = x$

Differentiate w.r.t. x

$f'(g(x)) \cdot g'(x) = 1$

$\therefore \frac{1}{1+(g(x))^4} \cdot g'(x) = 1$

$g'(x) = 1 + [g(x)]^4$

8. The inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$ is

(A) $-\frac{1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 2 & -3 \end{bmatrix}$ (B) $-\frac{1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

(C) $-\frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$ (D) $-\frac{1}{3}\begin{bmatrix} -3 & 0 & 0 \\ -3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

8. **(B)**

$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix} |A| = -3$

$A^{-1} = \frac{1}{|A|} \text{Adj } A$

$= \frac{1}{-3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

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9. If $\int \frac{1}{\sqrt{9-16x^2}}dx = \alpha \sin^{-1}(\beta x)+c$ then $\alpha + \frac{1}{\beta} =$

(A) 1 (B) $\frac{7}{12}$ (C) $\frac{19}{12}$ (D) $\frac{9}{12}$

9. **(A)**

$$\int \frac{1}{\sqrt{9-16x^2}}dx = \alpha \sin^{-1}(\beta x)+c$$

$$\int \frac{1}{\sqrt{3^2-(4x)^2}}dx = \frac{1}{4}\sin^{-1}\left(\frac{4x}{3}\right)+c$$

$$\alpha = \frac{1}{4} \qquad \beta = \frac{4}{3}$$

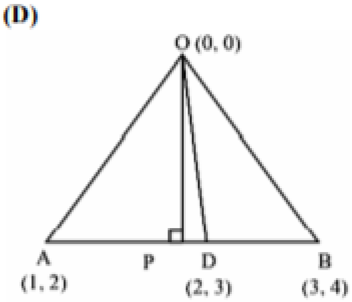
$$\alpha + \frac{1}{\beta} = \frac{1}{4}+\frac{3}{4} = 1$$

10. $O(0,0), A(1,2), B(3,4)$ are the vertices of ΔOAB. The joint equation of the altitude and median drawn from O is

(A) $x^2+7xy-y^2=0$ (B) $x^2+7xy+y^2=0$

(C) $3x^2-xy-2y^2=0$ (D) $3x^2+xy-2y^2=0$

10. **(D)**



Equation of median OD = y = mx $\Rightarrow 3 = 2m$

$\Rightarrow x = \frac{2}{3}$

$\therefore y = \frac{3}{2}x \Rightarrow 3x-2y=0$

Slope of AB $= \frac{2}{2} = 1 \Rightarrow$ Slope of OP = – 1

Equation of OP $\Rightarrow y=-x \Rightarrow x+y=0$

Joint equation of OP and OD $\Rightarrow (x+y)(3x-2y)=0$

$\Rightarrow 3x^2+xy-2y^2=0$

11. If the function $f(x)-\left[\tan \pi/4+x\right]^{1/x}$ for $x \neq 0$

$= K$ for $x = 0$

Is continuous at $x = 0$ then K= ?

(A) e (B) e^{-1} (C) e^{2} (D) e^{-2}

11. **(C)**

$$f(0) = \lim_{x\to 0} f(x)$$

$$= \lim_{x\to 0}\left[\tan\left(\frac{\pi}{4}+\pi\right)\right]^{\frac{1}{x}}$$

$$= \lim_{x\to 0}\left[\left(\frac{1+\tan x}{1-\tan x}\right)\right]^{\frac{1}{x}}$$

$$= \lim_{x\to 0}\frac{\left[(1+\tan \pi)\frac{1}{\tan x}\right]^{\frac{\tan x}{x}}}{\left[(1-\tan x)^{-\frac{1}{\tan x}}\right]^{\frac{\tan x}{x}}}$$

Taking limits

$$= \frac{e^{1}}{e^{-1}} = e^{1}\cdot e^{1} = e^{2}$$

12. For a invertible matrix A if A(adj A) = $\begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix}$ then $|A|$

(A) 100 (B) – 100 (C) 10 (D) – 10

12. **(C)**

$$A\,(\text{adj } A) = \begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix} = 10\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = 10\,I$$

We know that A (adj A) = $|A|I$

$\Rightarrow |A| = 10$

13. The solution of the differentiable equation $\frac{dy}{dx} = \tan\left(\frac{y}{x}\right)+\frac{y}{x}$ is

(A) $\cos\left(\frac{y}{x}\right) = cx$ (B) $\sin\left(\frac{y}{x}\right) = cx$

(C) $\cos\left(\frac{y}{x}\right) = cy$ (D) $\sin\left(\frac{y}{x}\right) = cy$

13. **(B)**

$$\frac{dy}{dx} = \tan\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)$$

$$\frac{y}{x} = v$$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ the given equation becomes

$$v + x\frac{dv}{dx} = \tan v + v$$

$$\frac{1}{\tan v}dv = \frac{1}{x}dx$$

$$\int \cot v\, dv = \int \frac{1}{x}dx$$

$$\log|\sin v| = \log x + \log c$$

$$= \log(xc)$$

$$\sin v = xc$$

$$\sin\left(\frac{y}{x}\right) = xc$$

14. In ΔABC if $\sin^2 A + \sin^2 B = \sin^2 C$ and $l(AB) = 10$ then the maximum value of the area of ΔABC is

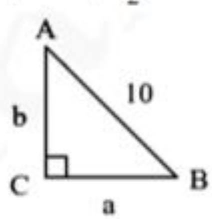
(A) 50 (B) $10\sqrt{2}$ (C) 25 (D) $25\sqrt{2}$

14. **(C)**

$\sin^2 A + \sin^2 B = \sin^2 C$

$\Rightarrow a^2 + b^2 = c^2$ (Sine Rule)

$A(\Delta ABC) = \frac{1}{2}ab \qquad \ldots(1)$



From sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{10}{1}$

$\Rightarrow a = 10\sin A, b = 10\sin B$

Using equation (1) $A(\Delta ABC) = \frac{1}{2}(10\sin A)(10\sin B)$

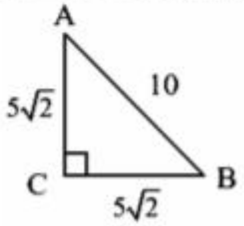
$= 50\sin A\sin B$

But maximum value of $\sin A\sin B = \frac{1}{2}$

$\therefore$ Maximum value of $A(\Delta ABC) = 50 \times \frac{1}{2} = 25$

OR

$\angle C = 90^0 \Rightarrow ABC$ is right angled triangle



$\therefore$ Area of Δ is maximum when it is $45^\circ - 45^\circ - 90^\circ\ \Delta$.

$\therefore A(\Delta ABC) = \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2} = 25$

15. If $x = f(t)$ and $y = g(t)$ are differentiable functions of t then $\frac{d^2y}{dx^2}$ is

(A) $\frac{f'(t).g''(t) - g'(t).f''(t)}{[f'(t)]^3}$ (B) $\frac{f'(t).g''(t) - g'(t).f''(t)}{[f'(t)]^2}$

(C) $\frac{g'(t).f''(t) - f'(t).g''(t)}{[f'(t)]^3}$ (D) $\frac{g'(t).f''(t) + f'(t).g''(t)}{[f'(t)]^3}$

15. **(A)**

$x = f(t)$

$y = g(t)$

$\frac{dx}{dt} = f'(t)$

$\frac{dy}{dt} = g'(t)$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{g'(t)}{f'(t)}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{g'(t)}{f'(t)}\right)$

$= \frac{f'(t)\cdot g''(t) - g'(t)\cdot f''(t)}{(f'(t))^2} \cdot \frac{dt}{dx}$

$= \frac{f'(t)\cdot g''(t) - g'(t)\cdot f''(t)}{(f'(t))^2} \cdot \frac{1}{f'(t)}$

$= \frac{f'(t)\cdot g''(t) - g'(t) f''(t)}{(f'(t))^3}$

16. The equation of line equally inclines to co-ordinate axes and passing through $(-3,2,-5)$ is

(A) $\frac{x+3}{1}=\frac{y-2}{1}=\frac{z+5}{1}$ (B) $\frac{x+3}{-1}=\frac{y-2}{1}=\frac{5+z}{-1}$

(C) $\frac{x+3}{-1}=\frac{y-2}{1}=\frac{z+5}{1}$ (D) $\frac{x+3}{-1}=\frac{2-y}{1}=\frac{z+5}{-1}$

16. **(A)**

Equation of line passing through (x_1, y_1, z_1) and having d.c.s. ℓ, m, n is

$$\frac{x-x_1}{\ell}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$$

Here $(x_1, y_1, z_1) = (-3, 2, -5)$

Also line is equally inclined to co-ordinate axes.

$\therefore \ell = -1, m = 1, n = -1$

$\therefore$ Equation of line is $\frac{x+3}{-1}=\frac{y-2}{1}=\frac{z+5}{-1}$

17. If $\int_0^{\pi/2} \log\cos x\,dx = \frac{\pi}{2}\log\left(\frac{1}{2}\right)$ then $\int_0^{\pi/2} \log\sec x\,dx =$

(A) $\frac{\pi}{2}\log\left(\frac{1}{2}\right)$ (B) $1-\frac{\pi}{2}\log\left(\frac{1}{2}\right)$ (C) $1+\frac{\pi}{2}\log\left(\frac{1}{2}\right)$ (D) $\frac{\pi}{2}\log 2$

17. **(D)**

$$\int_0^{\pi/2} \log\sec x dx = \int_0^{\pi/2} \log\left(\frac{1}{\cos x}\right)dx$$

$$= -\int_0^{\pi/2} \log(\cos x)dx$$

$$= -\frac{\pi}{2}\log(1/2) \quad \left(\log\left(\frac{1}{a}\right) = -\log a\right) \quad = \frac{\pi}{2}\log 2$$

18. A boy tosses fair coin 3 times if he gets ' 2X for X heads then his expected gain equals to ' ......

(A) 1 (B) $\frac{3}{2}$ (C) 3 (D) 4

18. **(C)**

For x heads, he gets y = 2x

x	0	1	2	3
y	0	2	4	6
p(y)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Expected gain = $\Sigma y_i p_i$

$$= 0\left(\frac{1}{8}\right)+2\left(\frac{3}{8}\right)+4\left(\frac{3}{8}\right)+6\left(\frac{1}{8}\right)=\frac{6+12+6}{8}=\frac{3}{2}$$

19. Which of the following statement pattern is a tautology?

(A) $p \vee (q \to p)$ (B) $\sim q \to \sim p$

(C) $(q \to p) \vee (\sim p \leftrightarrow q)$ (D) $p \wedge \sim p$

19. **(C)**

It can be done using truth table or using rules of logic.

(A) $p \vee (q \to p) \equiv p \vee (\sim q \vee p) \equiv p \vee p \vee \sim q$

$\equiv p \vee \sim q$

(B) $\sim q \to \sim p \equiv q \vee \sim p$

(D) $p \wedge \sim p \equiv F$

So left is (C)

(C)

p	q	$q \to p$	$\sim p$	$\sim p \leftrightarrow q$	$(q \to p) \vee (\sim p \leftrightarrow q)$
T	T	T	F	F	T
T	F	T	F	T	T
F	T	F	T	T	T
F	F	T	T	F	T

20. If the angle between the planes $\bar{r}.(m\hat{i} - \hat{j} + 2\hat{k}) + 3 = 0$ and $\bar{r}.(2\hat{i} - m\hat{j} - \hat{k}) - 5 = 0$ is $\pi/3$ then m =

(A) 2 (B) ±3 (C) 3 (D) – 2

20. **(C)**

Direction ratios $\bar{n}_1$ are m, –1, 2

Direction ratios $\bar{n}_2$ are 2, –m, –1

$\theta = \frac{\pi}{3}$

$\cos\theta = \left|\frac{\bar{n}_1 \cdot \bar{n}_2}{|\bar{n}_1||\bar{n}_2|}\right| \Rightarrow \frac{1}{2} = \left|\frac{2m + m - 2}{\sqrt{m^2+5}\sqrt{m^2+5}}\right|$

$\frac{1}{2} = \left|\frac{3m-2}{m^2+5}\right| \Rightarrow m^2 + 5 = \pm(6m - 4)$

$\Rightarrow m^2 + 5 = 6m - 4,\ m^2 + 5 = -6m + 4$

$m^2 - 6m + 9 = 0,\ m^2 + 6m + 1 = 0$

$(m - 3)^2 = 0$

$m = 3$

21. If the origin and the points $P(2,3,4), Q(1,2,3)$ and $R(x,y,z)$ are co-planer then

(A) $x - 2y - z = 0$ (B) $x + 2y + z = 0$

(C) $x - 2y + z = 0$ (D) $2x - 2y + z = 0$

21. **(C)**

O, P, Q, R are co–planar

$[\overline{OR}\ \ \overline{OP}\ \ \overline{OQ}] = 0$

$$\begin{vmatrix} x & y & z \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$x(9-8) - y(6-4) + z(4-3) = 0$

$x - 2y + z = 0$

Alternative

Points P, Q satisfy equations given in option.

22. If lines represented by equation $px^2 - qy^2 = 0$ are distinct then

(A) $pq > 0$ (B) $pq < 0$ (C) $pq = 0$ (D) $p + q = 0$

22. **(A)**

$px^2 - qy^2 = 0$

$a = p, b = -q, h = 0$

lines are real and distinct is $h^2 - ab > 0$

$\Rightarrow 0 + pq > 0$

$pq > 0$

23. Let □PQRS be a quadrilateral. If m and n are the midpoints of the sides PQ and RS respectively then $\overline{PS} + \overline{QR} =$

(A) $3\overline{MN}$ (B) $4\overline{MN}$ (C) $2\overline{MN}$ (D) $2\overline{MN}$

23. **(C)**

$\overline{m} = \dfrac{\overline{p} + \overline{q}}{2}$

$\overline{n} = \dfrac{\overline{r} + \overline{s}}{2}$

$\overline{PS} + \overline{QR} = \overline{s} - \overline{p} + \overline{r} - \overline{q}$

$= (\overline{r} + \overline{s}) - (\overline{p} + \overline{q})$

$= 2\overline{n} - 2\overline{m}$

$= 2(\overline{n} - \overline{m})$

$= 2\,\overline{MN}$

24. If slopes of lines represented by $Kx^2 + 5xy + y^2 = 0$ differ by 1 then K =

(A) 2 (B) 3 (C) 6 (D) 8

24. **(C)**

$kx^2 + 5xy + y^2 = 0$

$m_1 + m_2 = -5, m_1 m_2 = k, m_1 - m_2 = 1$

$(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1 m_2 \Rightarrow 1 = 25 - 4k \Rightarrow k = 6$

25. If vector $\overline{r}$ with d.c.s l, m, n is equally inclined to the co-ordinate axes , then the total number of such vector is

(A) 4 (B) 6 (C) 8 (D) 2

25. **(D)**

$$\bar{r} = |\bar{r}|\left(\pm\frac{1}{\sqrt{3}}\hat{i} \pm \frac{1}{\sqrt{3}}\hat{j} \pm \frac{1}{\sqrt{3}}\hat{k}\right)$$

For equally inclined to co–ordinate axes.

$\alpha = \beta = \gamma$

$\ell = m = n$

$\ell^2 + m^2 + n^2 = 1$

$3\ell^2 = 1$

$\ell^2 = \frac{1}{3}$

$\ell = \pm\frac{1}{\sqrt{3}} = m = n \Rightarrow \ell$, m, n each has 2 choices.

$\therefore$ total lines $= 2^3$

26. If $\int \frac{1}{(x^2+4)(x^2+9)}dx = A\tan^{-1}\frac{x}{2} + B\tan^{-1}\left(\frac{x}{3}\right) + C$ then $A - B =$

(A) $\frac{1}{6}$ (B) $\frac{1}{30}$ (C) $-\frac{1}{30}$ (D) $-\frac{1}{6}$

26. **(A)**

$$\frac{1}{AB} = \frac{1}{B-A}\left(\frac{1}{A} - \frac{1}{B}\right)$$

$$\int \frac{1}{(x^2+4)(x^2+9)}dx = \int \frac{1}{5}\left(\frac{1}{x^2+4} - \frac{1}{x^2+9}\right)dx$$

$$= \frac{1}{5}\left[\frac{1}{2}\tan^{-1}\frac{x}{2} - \frac{1}{3}\tan^{-1}\frac{x}{3}\right] + C$$

$$A = \frac{1}{10} \qquad B = -\frac{1}{15}$$

$$A - B = \frac{1}{10} + \frac{1}{15} = \frac{5}{30} = \frac{1}{6}$$

27. If α and β are roots of the equation $x^2 + 5|x| - 6 = 0$ then the value of $|\tan^{-1}\alpha - \tan^{-1}\beta|$ is

(A) $\frac{\pi}{2}$ (B) 0 (C) π (D) $\frac{\pi}{4}$

27. **(A)**

$x^2+5|x|-6=0$

$|x|^2+5|x|-6=0$

$|x|^2+6|x|-|x|-6=0$

$|x|(|x|+6)-1(|x|+6)=0$

$(|x|-1)(|x|+6)=0$

$|x|=1 \quad |x|\neq -6$ **(since modulus can not be giving negative values)**

$\therefore \quad |x|=1$

$\therefore \quad x=\pm 1$

$\alpha=1, \beta=-1$

$\therefore \quad |\tan^{-1}\alpha-\tan^{-1}\beta| = |\tan^{-1}1-\tan^{-1}(-1)|$

$=\left|\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)\right|$

$=\left|\frac{\pi}{2}\right|$

28. If $x=a\left(t-\frac{1}{t}\right), y=a\left(t+\frac{1}{t}\right)$ where t be the parameter then $\frac{dy}{dx}=?$

(A) $\frac{y}{x}$ (B) $\frac{-x}{y}$ (C) $\frac{x}{y}$ (D) $\frac{-y}{x}$

28. **(C)**

$x = a\left(t-\frac{1}{t}\right), \qquad y = a\left(t+\frac{1}{t}\right)$

$y^2-x^2 = a^2\left[\left(t+\frac{1}{t}\right)^2-\left(t-\frac{1}{t}\right)^2\right]$

$y^2-x^2 = 4a^2$

Differentiate w.r.t. x

$2y\frac{dy}{dx}-2x = 0$

$\therefore \quad \frac{dy}{dx} = \frac{x}{y}$

29. The point on the curve $y=\sqrt{x-1}$ where the tangent is perpendicular to the line $2x+y-5=0$ is

(A) $(2,-1)$ (B) $(10,3)$ (C) $(2,1)$ (D) $(5,-2)$

29. **(C)**

$\frac{dy}{dx} = \frac{1}{2\sqrt{x-1}} = m_1$

Slope of line $2x + y - 5 = 0$ is $m_2 = -2$

For lines are perpendicular

$m_1 m_2 = -1$

$\left(\frac{1}{2\sqrt{x-1}}\right)(-2) = -1$

$\frac{2}{2\sqrt{x-1}} = 1$

$\sqrt{x-1} = 1$

Squaring both sides, $x - 1 = 1$

$x = 2$

$\therefore \; y = \sqrt{x-1}$

$= \sqrt{2-1}$

$= \sqrt{1}$

$y = 1$

$\therefore \; (2, 1)$

30. If $\int \sqrt{\frac{x-5}{x-7}}\, dx = A\sqrt{x^2-12x+38} + \log\left|x-6+\sqrt{x^2-12x+35}\right| + C$ then A =

(A) -1 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) 1

30. **(D)**

$$\int \sqrt{\frac{x-5}{x-7}}\, dx = \int \frac{x-5}{\sqrt{x^2-12x+35}}\, dx = \frac{1}{2}\int \frac{2x-10}{\sqrt{x^2-12x+35}}\, dx$$

$$= \frac{1}{2}\int \frac{2x-12+2}{\sqrt{x^2-12x+35}}\, dx$$

$$= \frac{1}{2}\int \frac{2x-12}{\sqrt{x^2-12x+35}}\, dx + \int \frac{dx}{\sqrt{x^2-12x+36-1}}$$

$$= \frac{1}{2} 2\sqrt{x^2-12x+35} + \int \frac{dx}{\sqrt{(x-6)^2-1}} + c_1$$

$$= \sqrt{x^2-12x+35} + \log\left|x-6+\sqrt{x^2-12x+35}\right| + c$$

A $= 1$

31. A r. v. $X \sim B(n, p)$. If values of mean and variance of X are 18 and 12 respectively then total number of possible values of X are

(A) 54 (B) 55 (C) 12 (D) 18

31. **(B)**

Mean = np = 18

Variance = npq = 12

$\frac{npq}{np} = \frac{12}{18}$

$q = \frac{2}{3}$

$p = 1 - q = 1 - \frac{2}{3}$

$p = \frac{1}{3}$

np = 18

$n\left(\frac{1}{3}\right) = 18$

$\boxed{n = 54}$

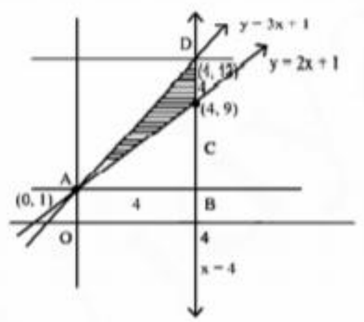
∴ values of X are
0, 1,2, ……… 54

∴ 55 values.

32. The area of the region bounded by the lines y = 2x + 1, y = 3x + 1 and x = 4 is

(A) 16 sq. unit (B) $\frac{121}{3}$sq.unit (C) $\frac{121}{6}$sq.unit (D) 8 sq. unit

32. **(D)**



A (Shaded region) = A(ΔABD) – A(ΔABC) = $\frac{1}{2}$[4 × 12 – 4 × 8] = $\frac{1}{2}$ (48 – 32) = 8 sq. units.

OR

$$A(\Delta ACD) = \frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ 4 & 9 & 1 \\ 4 & 13 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \times 16 = 8$$

33. A box contains 6 pens, 2 of which are defective. Two pens are taken randomly from the box. If r. v. X: Number of defective pens obtained, then standard deviation of X =

(A) $\pm\frac{4}{3\sqrt{5}}$ (B) $\frac{8}{3}$ (C) $\frac{16}{45}$ (D) $\frac{4}{3\sqrt{5}}$

33. **(D)**

x : no. of defective pens

Two pens are taken from box

$\therefore$ x can take values 0, 1, 2

$$p(x=0) = \frac{{}^4C_2}{{}^6C_2} = \frac{4\times3}{6\times5} = \frac{2}{5} = \frac{6}{15}$$

$$p(x=1) = \frac{{}^2C_1 \times {}^4C_1}{{}^6C_2} = \frac{2\times4\times2\times1}{6\times5} = \frac{8}{15}$$

$$p(x=2) = \frac{{}^2C_2}{{}^6C_2} = \frac{1\times2\times1}{6\times5} = \frac{1}{15}$$

x	p	x_ip_i	$x_i^2p_i$
0	$\frac{6}{15}$	0	0
1	$\frac{8}{15}$	$\frac{8}{15}$	$\frac{8}{15}$
2	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{4}{15}$

$$E(x) = \frac{10}{15} = \frac{2}{3}$$

$$E(x^2) = \frac{12}{15} = \frac{4}{5}$$

$$\text{Standard deviation} = \sqrt{E(x^2) - [E(x)]^2}$$

$$\text{Standard devivation} = \sqrt{\left(\frac{4}{5}\right) - \left(\frac{2}{3}\right)^2} = \sqrt{\frac{4}{5} - \frac{4}{9}} = \sqrt{\frac{4\times4}{45}} = \frac{4}{3\sqrt{5}}$$

34. If the volume of spherical ball is increasing at the rate of 4π cc/sec then the rate of change of its surface are when the volume is 288π cc is

(A) $\frac{4}{3}\pi cm^2/sec$ (B) $\frac{2}{3}\pi cm^2/sec$ (C) $4\pi cm^2/sec$ (D) $2\pi cm^2/sec$

34. **(A)**

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

When $V = 288\pi$

$$288\pi = \frac{4}{3}\pi r^3 \Rightarrow r = 6$$

$$\frac{dv}{dt} = 4\pi$$

$$\therefore 4\pi r^2 \frac{dr}{dt} = 4\pi = \frac{dr}{dt} = \frac{1}{r^2}$$

$$A = \text{Surface area} = 4\pi r^2$$

$$\therefore \frac{dA}{dt} = 8\pi r \frac{dr}{dt} = 8\pi r \times \frac{1}{r^2} = \frac{8\pi}{r} = \frac{8\pi}{6} = \frac{4\pi}{3}$$

35. If $f(x) = \log(\sec^2 x)\cot^2 x$ for $x \neq 0$

$= K$ for $x = 0$

is continuous at $x = 0$ then K is

(A) e^{-1} (B) 1 (C) e (D) 0

35. **(B)**

$$f(0) = \lim_{x\to 0} \log(\sec^2 x)^{\cot 2x}$$

$$k = \lim_{x\to 0} \cot^2 x \cdot \log(1+\tan^2 x)$$

$$= \lim_{x\to 0} \frac{\log(1+\tan^2 x)}{\tan^2 x}$$

$$k = 1$$

36. If c denotes the contradiction then dual of the compound statement $\sim p \wedge (q \vee c)$ is

(A) $\sim p \vee (q \wedge t)$ (B) $\sim p \wedge (q \vee t)$ (C) $p \vee (\sim q \wedge t)$ (D) $\sim p \vee (\sim q \wedge e)$

36. **(A)**

Dual of $\sim P \wedge (q \vee c) = \sim P \vee (q \wedge t)$

37. The differential equation of all parabolas whose axis is y – axis is

(A) $x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$ (B) $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

(C) $\frac{d^2y}{dx^2} - y = 0$ (D) $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

37. **(A)**

axis = y axis

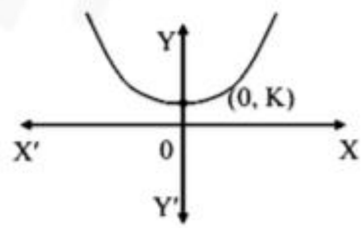
vertex is (0, k)

Equation of parabola is

$(x-0)^2 = 4a(y-k)$

$x^2 = 4ay - 4ak$

Differentiate w.r.t x



$$2x = 4a\frac{dy}{dx}$$

$$x = 2a\frac{dy}{dx}$$

$$\therefore \quad \frac{1}{2a} = \frac{1}{x}\frac{dy}{dx}$$

Differentiate w.r.t x,

$$\frac{d}{dx}\left(\frac{1}{x}\cdot\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{2a}\right)$$

$$\frac{1}{x}\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(-\frac{1}{x^2}\right) = 0$$

$$\therefore \quad x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$$

38. $\int_0^3 [x]dx =$ __________, where [x] is greatest integer function.

(A) 3 (B) 0 (C) 2 (D) 1

38. (A)

$$\int_0^3 [x]dx = \int_0^1 0\,dx + \int_1^2 1\,dx + \int_2^3 2\,dx$$

$$= [x]_1^2 + 2[x]_2^3$$

$$= (2-1) + 2(3-2)$$

$$= 1 + 2$$

$$= 3$$

39. The objective function of LPP defined over the convex set attain its optimum value at

(A) At least two of the corner points (B) All the corner points

(C) At least one of the corner points (D) None of the corner points

39. (C)

40. If the inverse of the matrix $\begin{bmatrix} \alpha & 14 & -1 \\ 2 & 3 & 1 \\ 6 & 2 & 3 \end{bmatrix}$ does not exist then the value of α is

(A) 1 (B) –1 (C) 0 (D) –2

40. (D)

$$A = \begin{bmatrix} \alpha & 14 & -1 \\ 2 & 3 & 1 \\ 6 & 2 & 3 \end{bmatrix}$$

$|A| = 7\alpha + 14$

A^{-1} does not exists if $|A| = 0$

$\Rightarrow 7\alpha + 14 = 0 \Rightarrow \alpha = -2$

41. If $f(x) = x$ for $x \le 0$
$= 0$ for $x > 0$ then $f(x)$ at $x = 0$ is

(A) Continuous but not differentiable
(B) Not continuous but differentiable
(C) Continuous and differentiable
(D) Not continuous and not differentiable

41. **(A)**

Continuity at $x = 0$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0} x = 0$

$\lim_{x \to 0^+} f(x) = 0$

$f(0) = 0$

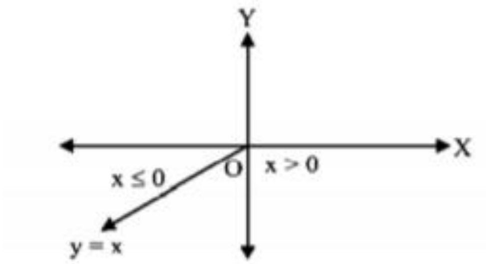
$\therefore$ continuous at $x = 0$

For differentiable

$f'(x) = 1 \quad x \le 0$
$\quad\;\; = 0 \quad x > 0$

$\therefore$ not differentiable

Alternative



It has sharp edge at $x = 0$

$\therefore$ not differentiable but continuous

42. The equation of the plane through (–1, 1, 2), whose normal makes equal acute angles with co – ordinates axes is

(A) $\bar{r}.(\hat{i}+\hat{j}+\hat{k}) = 2$
(B) $\bar{r}.(\hat{i}+\hat{j}+\hat{k}) = 6$
(C) $\bar{r}.(3\hat{i}-3\hat{j}+3\hat{k}) = 2$
(D) $\bar{r}.(\hat{i}-\hat{j}+\hat{k}) = 3$

42. **(A)**

Equation plane passing through

$A(\bar{a})$ & $\perp$ to $\bar{n}$ is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Here $\bar{a} = -\hat{i}+\hat{j}+2\hat{k}$, $\bar{n} = \hat{i}+\hat{j}+\hat{k}$

$\therefore$ $\bar{r}.(\hat{i}+\hat{j}+\hat{k}) = (-\hat{i}+\hat{j}+2\hat{k}).(\hat{i}+\hat{j}+\hat{k}) = 2$

43. Probability that a person will develop immunity after vaccination is .8. If 8 people are given the vaccine then probability that all develop immunity us =

(A) $(0.2)^8$ (B) $(0.8)^8$ (C) 1 (D) ${}^8C_6\,(0.2)^6\,(0.8)^2$

43. **(B)**

$(0.8)^8$

44. If the distance of points $2\hat{i}+3\hat{j}+\lambda\hat{k}$ from the plane $\bar{r}.(3\hat{i}+2\hat{j}+6\hat{k}) = 13$ is 5 units then λ =

(A) $6, -\frac{17}{3}$ (B) $6, \frac{17}{3}$ (C) $-6, -\frac{17}{3}$ (D) $-6, \frac{17}{3}$

44. **(A)**

Equation of plane $\bar{r}.(3\hat{i}+2\hat{j}+6\hat{k}) = 13$

i.e. $3x + 2y + 62 - 13 = 0$

Given point $(2, 3, \lambda)$

distance of plane from the point $= \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2+b^2+c^2}}\right|$

$$5 = \left|\frac{3(2)+2(3)+6\lambda-13}{\sqrt{9+4+36}}\right|$$

$$\therefore \quad 5 = \left|\frac{|6\lambda-1|}{7}\right|$$

$\Rightarrow$ $6\lambda - 1 = \pm 35$

$\Rightarrow$ $6\lambda = 36,\ 6\lambda = -34$

$\lambda = 6, \quad \lambda = -\frac{17}{3}$

45. The value of $\cos^{-1}\left(\cot\left(\frac{\pi}{2}\right)\right) + \cos^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ is

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) π

45. **(A)**

$$\cos^{-1}\left(\cot\frac{\pi}{2}\right) + \cos^{-1}\left(\sin\frac{2\pi}{3}\right) = \cos^{-1}(0) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\pi}{2} + \cos^{-1}\left(\cos\frac{\pi}{6}\right)$$

$= \frac{\pi}{2}+\frac{\pi}{6}$

$= \frac{4\pi}{6}$

$= \frac{2\pi}{3}$

46. The particular solution of the differential equation $xdy + 2ydx = 0$, when $x = 2, y = 1$ is

(A) $xy = 4$ (B) $x^2y = 4$ (C) $xy^2 = 4$ (D) $x^2y^2 = 4$

46. (B)

$xdy + 2ydx = 0$

$\Rightarrow \frac{dy}{y} + \frac{2dx}{x} = 0$

on Integrating

$\int\frac{dy}{y} + 2\int\frac{dx}{x} = C_1$

$\log y + 2 \log x = \log C$

$\therefore x^2y = C$

When $x = 2, y = 1, C = 4$

$\therefore$ Particular solution is $x^2y = 4$

47. ΔABC has vertices at $A \equiv (2,3,5), B \equiv (-1,3,2)$ and $C \equiv (\lambda, 5, \mu)$. If the median through A is equally inclined to the axes, then the values of λ and μ respectively are

(A) 10, 7 (B) 9, 10 (C) 7, 9 (D) 7, 10

47. (D)

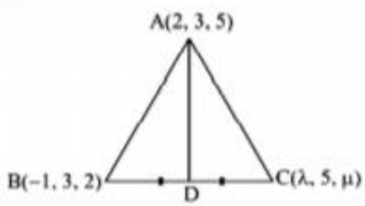
$D = \left(\frac{\lambda-1}{2}, \frac{5+3}{2}, \frac{\mu+2}{2}\right)$

$= \left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right)$

Direction ratios of AD are $\frac{\lambda-1}{2}-2, 4-3, \frac{\mu+2}{2}-5$

i.e. $\frac{\lambda-1-4}{2}, 1, \frac{\mu+2-10}{2}$

i.e. $\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}$



go by options (since the line AD is equally inclined to coordinate axes, its direction ratios are in ratio $\pm 1 : \pm 1 : \pm 1$)

48. For the following distribution function F(x) of a r.v. X

X	1	2	3	4	5	6
F(x)	0.2	0.37	0.48	0.62	0.85	1

$P(3 < x \le 5)$

(A) 0.48 (B) 0.37 (C) 0.27 (D) 1.47

48. (B)

x	1	2	3	4	5	6
f(x)	0.2	0.37	0.48	0.62	0.85	1
p(x)	0.2	0.17	0.11	0.14	0.23	0.15

$p(3 < x \le 5) = p(x = 4) + p(x = 5)$
$= 0.14 + 0.23$
$= 0.37$

49. The lines $\frac{x-1}{2} = \frac{y+1}{2} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect each other at point

(A) (–2, –4, 5) (B) (–2, –4, –5) (C) (2, 4, –5) (D) (2, –4, –5)

49. **(B)**

Go by options, only (B) option satisfies the first line.

50. $\int \frac{\sec^8 x}{\operatorname{cosec} x} dx =$

(A) $\frac{\sec^8 x}{8} + c$ (B) $\frac{\sec^7 x}{7} + c$ (C) $\frac{\sec^6 x}{6} + c$ (D) $\frac{\sec^9 x}{9} + c$

50. **(B)**

$$\int \frac{\sec^8 x}{\operatorname{cosec} x} dx = \int \frac{\sin x}{\cos^8 x} dx$$
$$= \int \tan x . \sec^7 x \, dx$$
$$= \int \sec^6 x . \sec x \tan x \, dx$$

$\sec x = t$

$dt = \sec x . \tan x . dx$

$$= \int t^6 dt$$
$$= \frac{t^7}{7} + c$$
$$= \frac{\sec^7 x}{7} + c$$